Electromagnetic Scattering from a Coaxial Dielectric Circular Cylinder Loading a Semicircular Gap in a Ground Plane

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***Abstract*—An exact dual series solution of a plane wave incident on a coaxial dielectric circular cylinder imbedded in a semicircular gap of a ground plane is presented. Both TM and TE cases are considered here. The scattered field is represented in terms of an infinite series of cylindrical waves with unknown coefficients. By applying the boundary conditions and employing the partial orthogonality of the trigonometric functions the scattering coefficients are obtained. The resulting infinite series is then truncated to a finite number of terms to produce numerical results. For the sake of comparison with the published data some special cases are introduced first. The comparisons showed excellent agreement in all cases.**

I. Introduction

The Radar Cross Section (RCS) of channels, grooves and cracks in a ground plane is of an interest to many investigators. Numerical techniques have been used to determine many characteristics of arbitrary grooves in a ground plane. For instance, Senior and Volakis [1] described a method for obtaining the scattered field from a narrow gap backed by a cavity. They suggested an equivalent problem of a narrow resistive strip inserted in a perfectly conducting plane. The scattered field of the original problem can be obtained by replacing the resistance R by η which is assumed to be the impedance looking into the cavity. Another approach was introduced by Senior *et al.* [2] based on the equivalence principles to develop coupled integral equations for electric and magnetic currents which exist in cavity walls and apertures. The equations were then solved by the method of moments.

Reduction of RCS from conducting structure is also became the subject of many investigations. Karunaratne *et al.* [3] and [4] studied the TM and TE scattering from a conducting strip loaded by a dielectric cylinder. The electromagnetic backscattering by a dielectric cylinder partially embedded a ground plane has also been examined by Kolbehdari *et al.* [5]. They utilize the equivalence theorem to obtain an integral equation for the equivalent magnetic current on the dielectric interface which is solved using Galerkin's method. Their solution can be classified as a numerical solution. On the other hand an analytical approximate solution to the same problem is also presented in [6]. Therefore an exact analytical solution to this problem is still needed.

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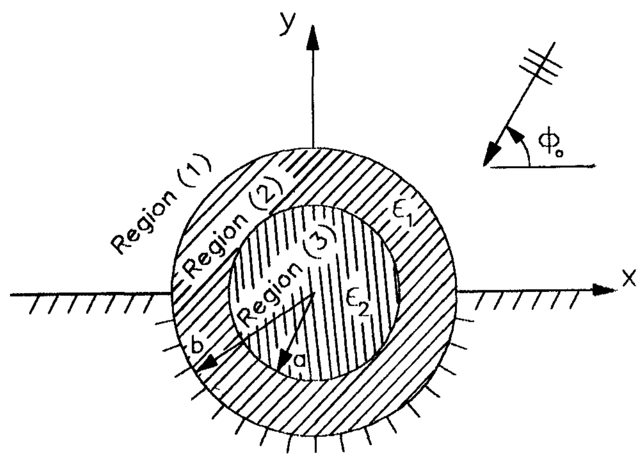


Fig. 1. Geometry of the problem.

Recently, an exact dual series solution of a plane wave scattering by a semicircular channel in a ground plane has been reported [7]. On the other hand, scattering by a coaxial dielectric or dielectric coated conducting cylinders has been extensively investigated [8]–[10]. It will be interest to use the exact method of [7] to obtain a solution for scattering by a coaxial dielectric cylinder loading a semicircular channel in a ground plane. The problem in this form becomes more general than any other geometry treated before. Also, it is inherent many special cases such as those of a conductor, an air or a plasma inner core. The study of the RCS of such a structure is an important topic which may be used as an antenna if a slot is opened in the conducting core.

II. Statement of the Problem

A two dimension cross-sectional area of two concentric dielectric cylinders embedded in a semicircular gap of a ground plane is illustrated in Fig. 1. The outer radius of the coating is denoted by "b" while the radius of the inner cylinder is denoted by "a," (ε_1, μ_1) and (ε_2, μ_2) are, respectively, their permittivities and permeabilities. The ground plane is assumed to be perfectly conducting. The structure is illuminated by a plane wave making an angle ϕ_o with respect to the x -axis of the coordinate system assumed at the center of the inner cylinder. The time dependence $e^{j\omega t}$ is assumed and omitted throughout.

A. TM Case

Consider the TM case, where the incident electric field has the z-component

$$E_z^{inc} = E_o\, e^{jkr\cos(\phi-\phi_o)} = E_o \sum_{n=-\infty}^{\infty} j^n J_n(kr) e^{jn(\phi-\phi_o)} \tag{1}$$

where k is the free space wavenumber $(2\pi/\lambda)$ and λ is the wave length. $J_n(x)$ is the Bessel function of first kind, order n and argument x. One should notice that all fields are expanded in terms of cylindrical functions.

The space above the ground plane is divided into three regions. The first region lies outside the coaxial cylinder while the second and third regions are inside the dielectric coating and the inner dielectric cylinder, respectively. The total electric field in region 1 can be expressed as the sum of incident, reflected and diffracted electric fields, i.e.

$$E_z^{(1)} = E_z^{inc} + E_z^{ref} + E_z^{Diff} = E_o \sum_{n=1}^{\infty} [4j^n J_n(kr) \sin n\phi_o + A_n^{TM} H_n^{(2)}(kr)] \cdot \sin n\phi \quad (r \geq b) \tag{2}$$

where A_n^{TM} are the unknown coefficients of the diffracted field. $H_n^{(2)}(x)$ is the Hankel function of the second kind, order n and argument x. The total electric field in regions 2 and 3 can be obtained by solving the Helmholtz wave equation which is expressed as an infinite series of cylindrical waves of unknown coefficients, i.e.

$$E_z^{(2)} = E_o \sum_{n=0}^{\infty} S_n(k_1 r)[B_n^{TM} \cos n\phi - C_n^{TM} \sin n\phi] \quad (a \leq r \leq b,\ C_o^{TM} = 0) \tag{3}$$

$$E_z^{(3)} = E_o \sum_{n=0}^{\infty} J_n(k_2 r)[B_n^{TM} \cos n\phi - C_n^{TM} \sin n\phi] \quad (0 \leq r \leq a,\ C_o^{TM} = 0) \tag{4}$$

where

$$S_n(k_1 r) = \tfrac{1}{2}\pi k_1 a[JY_n J_n(k_1 r) - JJ_n Y_n(k_1 r)] \tag{5}$$

$$JY_n = J_n(k_2 a) Y_n'(k_1 a) - \frac{\eta_1}{\eta_2} J_n'(k_2 a) Y_n(k_1 a) \tag{6}$$

$$JJ_n = J_n(k_2 a) J_n'(k_1 a) - \frac{\eta_1}{\eta_2} J_n'(k_2 a) J_n(k_1 a). \tag{7}$$

While for the conducting core

$$JY_n = Y_n(k_1 a) \quad \text{and} \quad JJ_n = J_n(k_1 a)$$

Here $k_1 = k\sqrt{\varepsilon_{r_1}\mu_{r_1}}$, $k_2 = k\sqrt{\varepsilon_{r_2}\mu_{r_2}}$, and $\eta = \sqrt{\mu/\varepsilon}$. $Y_n(x)$ is the Bessel function of the second kind, order n and argument x. Also any Bessel function with a prime denotes its derivative with respect to its argument. It is worth noting that the boundary conditions of the continuous tangential components of the electric and magnetic fields on the surface between the inner dielectric cylinder and the coating are satisfied in (3). In order to calculate the unknown coefficient in (2) and (3), the boundary conditions of the zero tangential electric field at $r = b$ and $\pi < \phi < 2\pi$ and continuous fields (i.e., E_z and H_ϕ) across the aperture $0 < \phi < \pi$ are applied to obtain

$$\sum_{n=0}^{\infty} B_n^{TM} S_n(k_1 b) \cos n\phi = \sum_{n=1}^{\infty} C_n^{TM} S_n(k_1 b) \sin n\phi \quad \pi < \phi < 2\pi \tag{8}$$

$$\sum_{n=0}^{\infty} B_n^{TM} S_n(k_1 b) \cos n\phi = \sum_{n=1}^{\infty} \{C_n^{TM} S_n(k_1 b) + 4j^n J_n(kb) \sin n\phi_o + A_n^{TM} H_n^{(2)}(kb)\} \cdot \sin n\phi \quad 0 < \phi < \pi \tag{9}$$

$$\sum_{n=0}^{\infty} B_n^{TM} S_n'(k_1 b) \cos n\phi = \sum_{n=1}^{\infty} \left\{ C_n^{TM} S_n'(k_1 b) + \frac{\eta_1}{\eta_o} [4j^n J_n'(kb) \sin n\phi_o + A_n^{TM} H_n^{(2)'}(kb)] \right\} \sin n\phi \quad 0 < \phi < \pi \tag{10}$$

where $S_n'(k_1 b)$ is the derivative of (5) with respect to $k_1 r$ at $r = b$. Changing $\phi \to \phi - \pi$ in (8), (8)–(10) can be written in the form

$$\sum_{n=0}^{\infty} g_n(kb) \cos n\phi = \sum_{n=1}^{\infty} f_n(k'b) \sin n\phi \quad 0 < \phi < \pi. \tag{11}$$

The partial orthogonal of the sinusoid of (11) over $\phi = 0$ to π is given by [7]

$$f_m(k'b) = \frac{4}{\pi} \sum_{\substack{n=0 \\ [(m+n)odd]}}^{\infty} \frac{m g_n(kb)}{m^2 - n^2} \quad m = 1, 2, 3, \cdots \tag{12}$$

where k' could be k or k_1. Employing (12) in (8)–(10) with the necessary mathematical manipulation, one can obtain

$$\frac{2j^{m-1} \sin m\phi_o S_m(k_1 b)}{mkb} = \sum_{\substack{n=0 \\ [(m+n)odd]}}^{\infty} \frac{B_n^{TM}}{m^2 - n^2} [S_n(k_1 b) G_m(k_1 b) + S_m(k_1 b) G_n(k_1 b)] \quad m = 1, 2, 3 \tag{13}$$

where

$$G_n(k_1 b) = S_n(k_1 b) H_m^{(2)'}(kb) - \frac{\eta_o}{\eta_1} S_n'(k_1 b) H_m^{(2)}(kb). \tag{14}$$

Equation (13) can be solved numerically to obtain the constants B_n^{TM}. The infinite series involved in the solution is convergent (which is illustrated in the results), therefore it will be truncated after a certain number of terms which depend on the largest argument of the Bessel function (i.e., $k_1 b$). Once the values of B_n^{TM} are calculated, the coefficients A_l^{TM} can be calculated from

$$A_l^{TM} = \frac{1}{H_l^{(2)}(kb)} \cdot \left\{ -4j^l J_l(kb) \sin l\phi_o + \frac{4}{\pi} \sum_{\substack{n=0 \\ [(l+n) odd]}}^{\infty} \cdot \frac{2l}{l^2 - n^2} S_n(k_1 b) B_n^{TM} \right\} \quad l = 1, 2, 3, \cdots. \tag{15}$$

B. TE Case

For the TE case, the incident magnetic field has the z-component

$$H_z^{inc} = H_o e^{jkr \cos(\phi - \phi_o)} = H_o \sum_{n=-\infty}^{\infty} j^n J_n(kr) e^{jn(\phi-\phi_o)}. \tag{16}$$

The total magnetic field in region 1 is composed of incident, reflected and diffracted fields, i.e.

$$H_z^{(1)} = H_z^{inc} + H_z^{ref} + H_z^{Diff} = H_o \sum_{n=0}^{\infty} [2\varepsilon_n j^n J_n(kr) \cos n\phi_o + A_n^{TE} H_n^{(2)}(kr)] \cdot \cos n\phi \quad (r \geq b) \tag{17}$$

where $\varepsilon_n = 1$ for $n = 0$ and 2 for $n > 0$ and A_n^{TE} are the unknown coefficients of the diffracted field. The total electric field in regions 2 and 3 are solutions of the Helmholtz wave equation in cylindrical coordinates, i.e.

$$H_z^{(2)} = H_o \sum_{n=0}^{\infty} S_n(k_1 r)[B_n^{TE} \cos n\phi - C_n^{TE} \sin n\phi] \quad (a \leq r \leq b,\ C_o^{TE} = 0) \tag{18}$$

$$H_z^{(3)} = H_o \sum_{n=0}^{\infty} J_n(k_2 r)[B_n^{TE} \cos n\phi - C_n^{TE} \sin n\phi] \quad (0 \leq r \leq a,\ C_o^{TE} = 0) \tag{19}$$

where $S_n(k_1 r)$ is given in (5) except η_1/η_2 is replaced by η_2/η_1 in (6) and (7). Again the expressions of the magnetic fields satisfy the boundary conditions of continuous tangential field components at the dielectric interface. The unknown scattering coefficients are then calculated from the boundary conditions of vanishing tangential components of E_ϕ at $r = b$ and $\pi < \phi < 2\pi$ and continuous H_z and E_ϕ at $r = b$ and $0 < \phi < \pi$. Enforcing these boundary conditions and following the same procedure used for the TM case, one obtains

$$\frac{2j^{m+1} \cos m\phi_o S_m'(k_1 b)}{kb} = \sum_{\substack{n=1 \\ [(m+n) odd]}}^{\infty} \frac{nC_n^{TE}}{n^2 - m^2} [S_n'(k_1 b) G_m(k_1 b) + S_m'(k_1 b) G_n(k_1 b)] \quad m = 0, 1, 2, \cdots \tag{20}$$

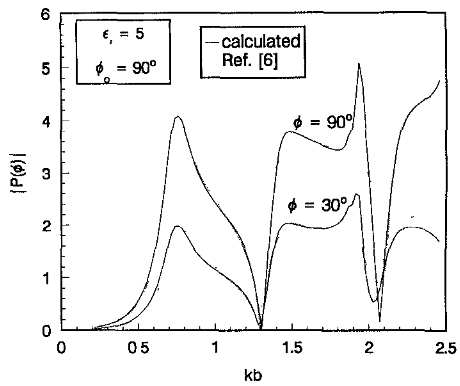


Fig. 2. Scattering pattern of a dielectric cylinder in a semi-circular channel.

where $G_n(k_1 b)$ is the same as (14) with η_o/η_1 replaced by η_1/η_o. The above equation can be solved numerically to obtain the constants C_n^{TE}. Once the values of C_n^{TE} are calculated the coefficients A_l^{TE} can be calculated from

$$A_l^{TE} = \frac{-2\varepsilon_l}{H_l^{(2)'}(kb)} \cdot \left\{ j^l J_l'(kb) \cos l\phi_o + \frac{\eta_1}{\pi\eta_o} \sum_{\substack{n=0 \\ [(l+n) odd]}}^{\infty} \cdot \frac{2n}{n^2 - l^2} S_n'(k_1 b) C_n^{TE} \right\} \quad l = 0, 1, 2, \cdots. \tag{21}$$

The diffracted field is then evaluated at a far point using the asymptotic expression of the Hankel function for a large argument, i.e.

$$\frac{E_z^{Diff}}{H_z^{Diff}} = \frac{E_o}{H_o} \sqrt{\frac{2}{\pi k\rho}} e^{-j(k\rho - \pi/4)} P(\phi) \tag{22}$$

where

$$P(\phi) = \sum_{n=1}^{\infty} j^n \frac{A_n^{TM}}{A_n^{TE}} \frac{\sin n\phi}{\cos n\phi}. \tag{23}$$

The scattering properties of two-dimensional bodies of infinite length are conveniently described in terms of the echo width, i.e.

$$W(\phi) = \frac{4}{k} |P(\phi)| \tag{24}$$

III. Numerical Results

To check the accuracy of our calculations, the special case of a dielectric cylinder loading a semicircular channel in a ground plane is introduced. In this case the radius of the dielectric core in our geometry is set to zero. The magnitude of the far scattered field pattern $P(\phi)$ is calculated and plotted versus kb at different values of ϕ as shown in Fig. 2. The parameters ε_r and ϕ_o are equal to 5 and 90°, respectively. Comparison between out results and their correspondence in

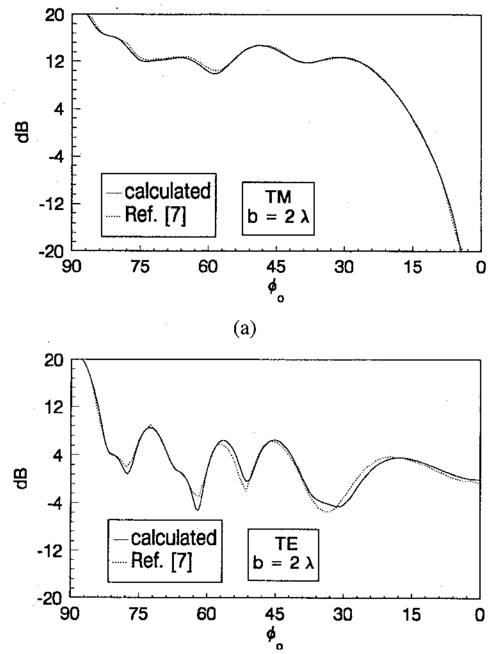


Fig. 3. Backscattering echo width versus ϕ_o for semi-circular channel.

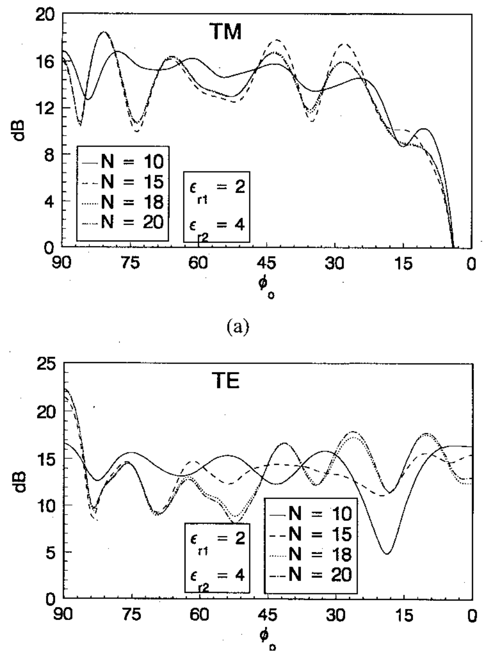


Fig. 4. Backscattering echo width versus ϕ_o for a coaxial cylinder of ($ka = 2\ \pi$, $kb = 4\pi$) for different values of N.

[6] showed an excellent agreement. Another special case of scattering by a semi-circular channel in the ground plane is considered. Both the dielectric constants of the core and the coating are set to ε_o in our geometry. The backscattering pattern for both TM and TE cases are calculated for a channel of radius 2λ. A comparison with the published data showed excellent agreement as illustrated in Fig. 3.

In order to verify that the infinite series involved in the solution is rapidly convergent, a general example is considered.

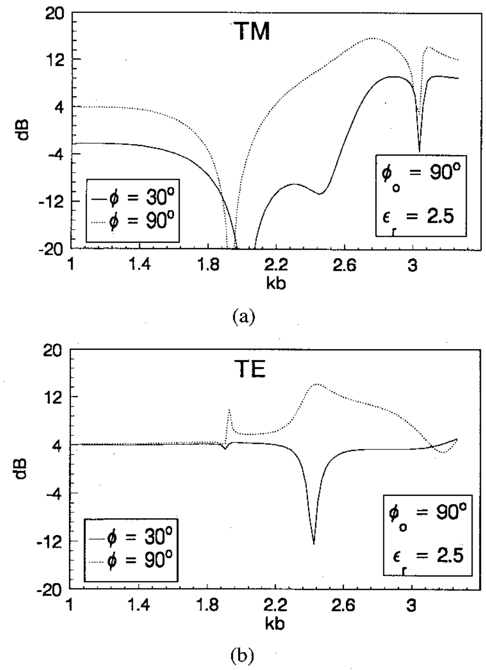


Fig. 5. Scattering echo width versus kb for conducting core ($ka = 1$).

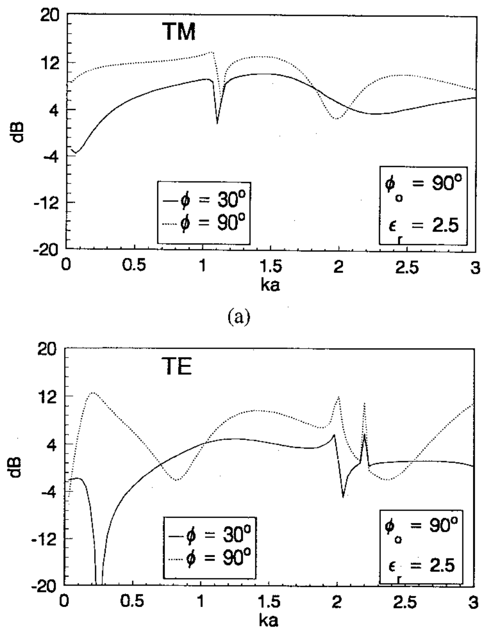
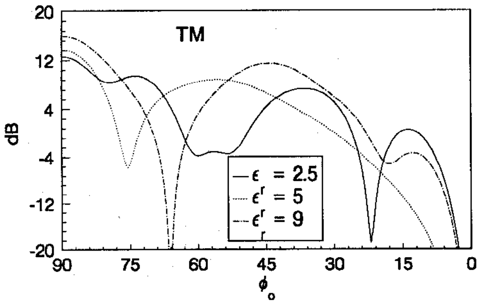


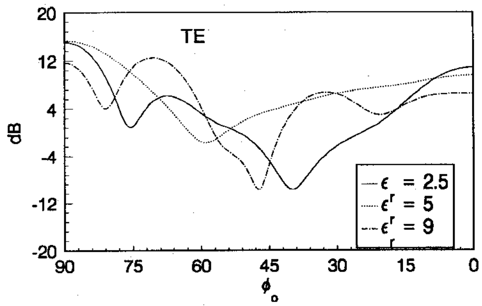
Fig. 6. Scattering echo width versus ka for conducting core ($kb = 3.14$).

The example shows the variation of the backscattering echo width for a coaxial dielectric cylinder of $a = 1\lambda$, $b = 2\ \lambda$, $\varepsilon_{r1} = 2$ and $\varepsilon_{r2} = 4$ with the number of terms of the infinite series N. As one can see from Fig. 4, the convergence is achieved after $N = 20$ for a maximum dimension of $2\ \lambda$. That is relatively a small number of terms required to achieve convergence.

Now, three groups of results are presented. The first group corresponds to a conducting core, where the echo width is calculated in three different cases. In the first case, the scattering echo widths versus the coating radius for $ka = 1.0$

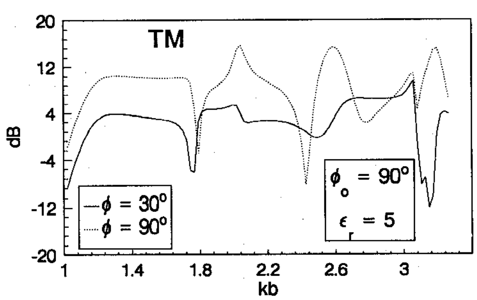


(a)



(b)

Fig. 7. Backscattering echo width versus ϕ_o for conducting core ($ka = 3, kb = 5$).



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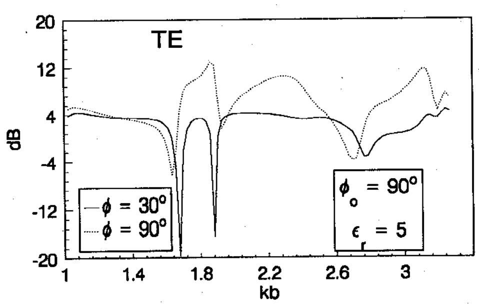
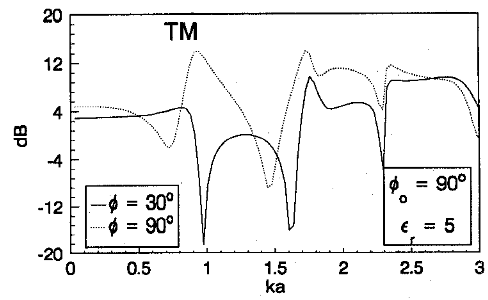


(b)

Fig. 8. Scattering echo width versus kb for air core ($ka = 1$).



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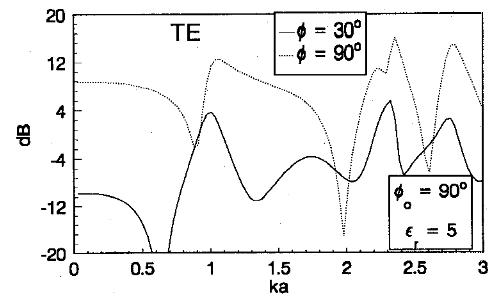
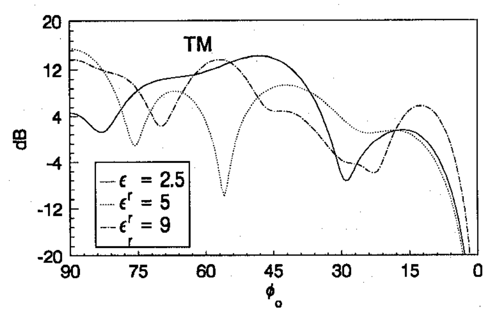
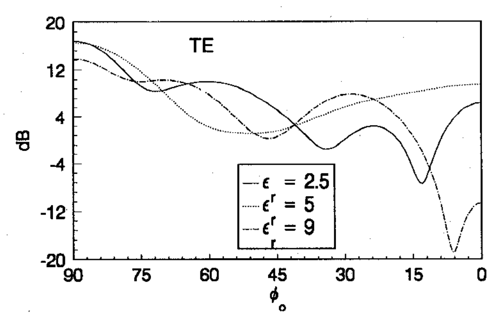


(b)

Fig. 9. Scattering echo width versus ka for air core ($kb = 3.14$).



(a)



(b)

Fig. 10. Backscattering echo width versus ϕ_o for air core ($ka = 3, kb = 5$).

are calculated at $\phi = 30°$ and $90°$ for both TM and TE cases as illustrated in Fig. 5. In the second case the outer radius of the dielectric coating is kept constant while the radius of the conducting core is varied from 0 to $ka = 3$. Plots of the echo width at two selected angles are shown in Fig. 6. The third case of this group displays the change in the backscattering echo with respect to the permittivity. Three different values of permittivity are considered, and the corresponding TM and TE backscattering echo widths are illustrated in Fig. 7.

For the second group the core is considered to be an air and cases similar to those taken for the conducting core are presented. Figs. 8–10, illustrate the scattering echo width versus kb, scattering echo width versus ka and backscattering echo width, respectively. All geometrical parameters are given in figure captions. The comparison between this group and the previous one shows a significant changes in the echo width due to the core material. These results are useful for predicting the core material without causing damage to the coated material.

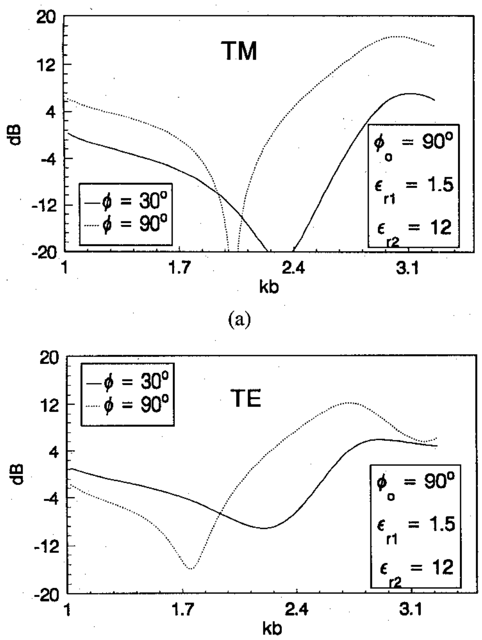


Fig. 11. Scattering echo width versus kb for dielectric core ($ka = 1$).

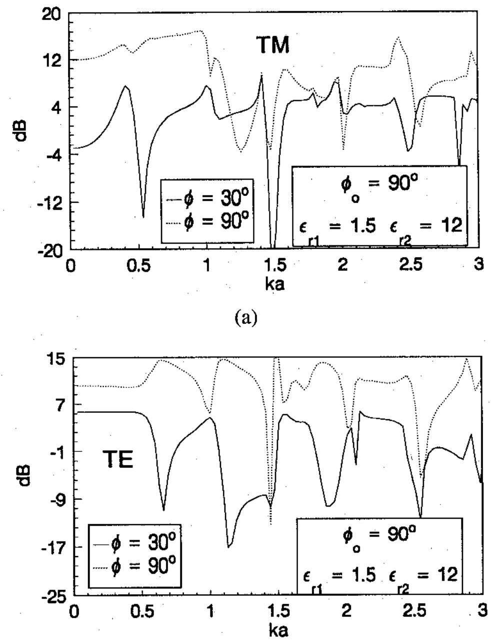


Fig. 12. Scattering echo width versus ka for dielectric core ($kb = 3.14$).

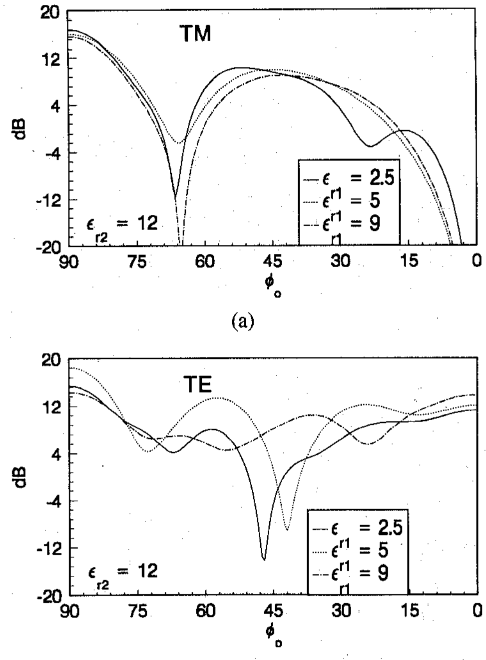


Fig. 13. Backscattering echo width versus ϕ_o for dielectric core ($ka = 3, kb = 5$).

The case of a dense core and light coating is then considered in the last group. The core and coating relative permittivities are considered as 12 and 1.5, respectively. The scattering echo width versus kb is illustrated in Fig. 11. One can see that the variation in echo width in this case is slower than the other corresponding cases. The change of echo width with ka while kb is kept constant at π is shown in Fig. 12. A fast change in the echo width with respect to the other cases is noticed. Finally the backscattering echo width is studied for different values of ε_{r1} as shown in Fig. 13. It is found that for the TM case there is no appreciable change in the echo width pattern. The scattering echo width patterns in general are important in target identification. Therefore the different patterns illustrated above are useful in this regard.

IV. Conclusion

An exact dual series solution based on the boundary value method for the problem of scattering by a coaxial dielectric circular cylinder in a semi-circular channel is introduced. Both TM and TE cases are investigated by using appropriate boundary conditions. The solution enables the study of scattering cross sections for several dielectric coatings which may be needed if the structure is part of a large target. The dielectrics are in our solution considered lossless, however lossy dielectrics can be considered but this requires additional research.

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